Test 4A - MTH 1410

Dr. Graham-Squire, Spring 2013

8

12:15 give

12:23 ⇒ 35 min.

8

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Unless otherwise stated, you should <u>use calculus</u> to justify your answers (in other words, just looking at a graph is NOT enough of a reason).
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are <u>not</u> allowed on the test. Calculators are allowed on the first 6 questions, but are <u>not</u> allowed on the last 2 questions of this test.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. Make sure you sign the pledge.
7. Number of questions = 4. Total Points = 40.

1. (10 points) Let $f(x)$ be given by the following graph:

(a) Use a Riemann sum to approximate the definite integral $\int_1^3 f(x)\,dx$. Use four subintervals and evaluate at the *right* endpoint (that is, find R_4).

(6) Approximately $0.5(1.8+1.5+1.3+1.5) = 0.5(6.1) = \boxed{3.05}$

(b) Do you think your approximation from (a) is an overestimate or an underestimate? Explain your reasoning.

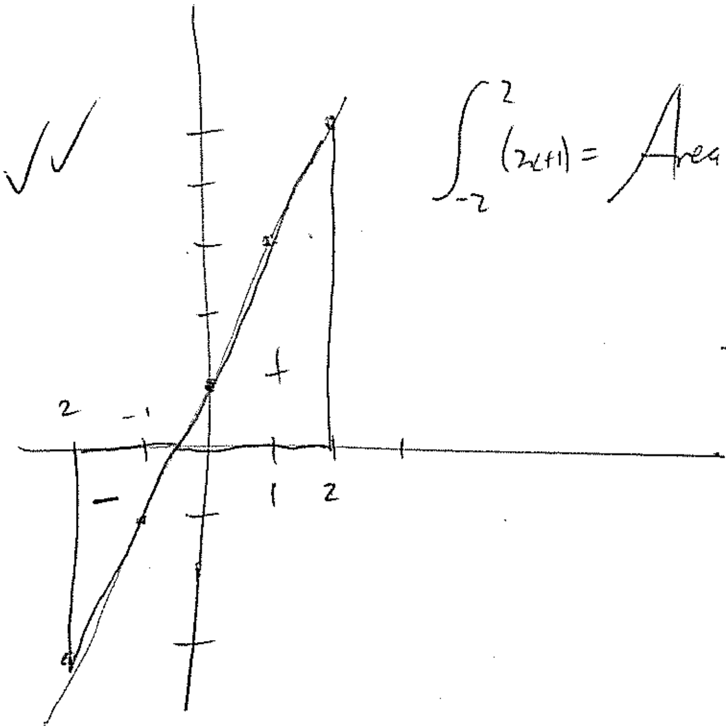
(2) Overestimate, but it is pretty close. The first rectangle is a large overestimate, More then the 2nd is an underestimate. The last two seem to cancel each other out.

(c) How could you improve your estimate for the value of $\int_1^3 f(x)\,dx$? Explain why this would improve the estimate (you do not actually have to improve the estimate, just explain what you would do and why).

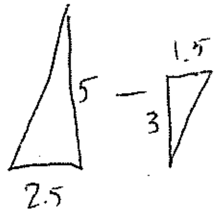
(2) Adding in more subintervals and calculating more rectangles would give less error and thus a better approximation.

or could do midpoint.

2. (10 points) Use formula(s) from geometry to find the exact value of $\int_{-2}^{2} (2x + 1)\, dx$. You can check your work by evaluating the definite integral using calculus, but you **must show how to get the answer from geometry** to get full credit.



$\int_{-2}^{2} (2x+1) = \text{Area} =$



$= \frac{1}{2}(5)(2.5) - \frac{1}{2}(3)(1.5)$

$6.25 - 2.25$

$\boxed{= 4}$

NO CALCULATORS

Name: Key

3. (10 points) Use calculus to evaluate the definite integrals:

(a) $\int_1^2 x(x^2+1)\,dx$

$= \int_1^2 (x^3 + x)\,dx$ ✓

✓✓ $\frac{1}{4}x^4 + \frac{x^2}{2}\Big|_1^2 = \frac{1}{4}(2^4) + \frac{2^2}{2} - \left(\frac{1}{4}(1) + \frac{1}{2}(1)\right)$ ✓

$= 4 + 2 - \frac{3}{4}$ ✓

$= 5.25$ or $5\frac{1}{4}$ or $\frac{21}{4}$.

(b) $\int_0^\pi (2^x + \sec x \tan x)\,dx$ ⟵ Simplify

$= \frac{2^x}{(\ln 2)} + \sec x \Big|_0^\pi$ ✓✓

$= \frac{2^\pi}{\ln 2} + \sec \pi - \left(\left(\frac{2^0}{\ln 2}\right) + \sec 0\right)$ ✓✓

$= \frac{2^\pi}{\ln 2} - 1 - \left(\frac{1}{\ln 2} + 1\right)$. ✓

$= \boxed{\frac{2^\pi - 1}{\ln 2} - 2}$ ✓

4. (10 points) Find the following indefinite integrals (i.e. the most general antiderivative):

(a) $\int \left(\frac{1}{\sqrt{1-x^2}} + e^x \right) dx$

$$\boxed{\sin^{-1} x + e^x + C}$$

(b) $\int \left(\frac{x^2 + x^7 + 3}{x^3} \right) dx$

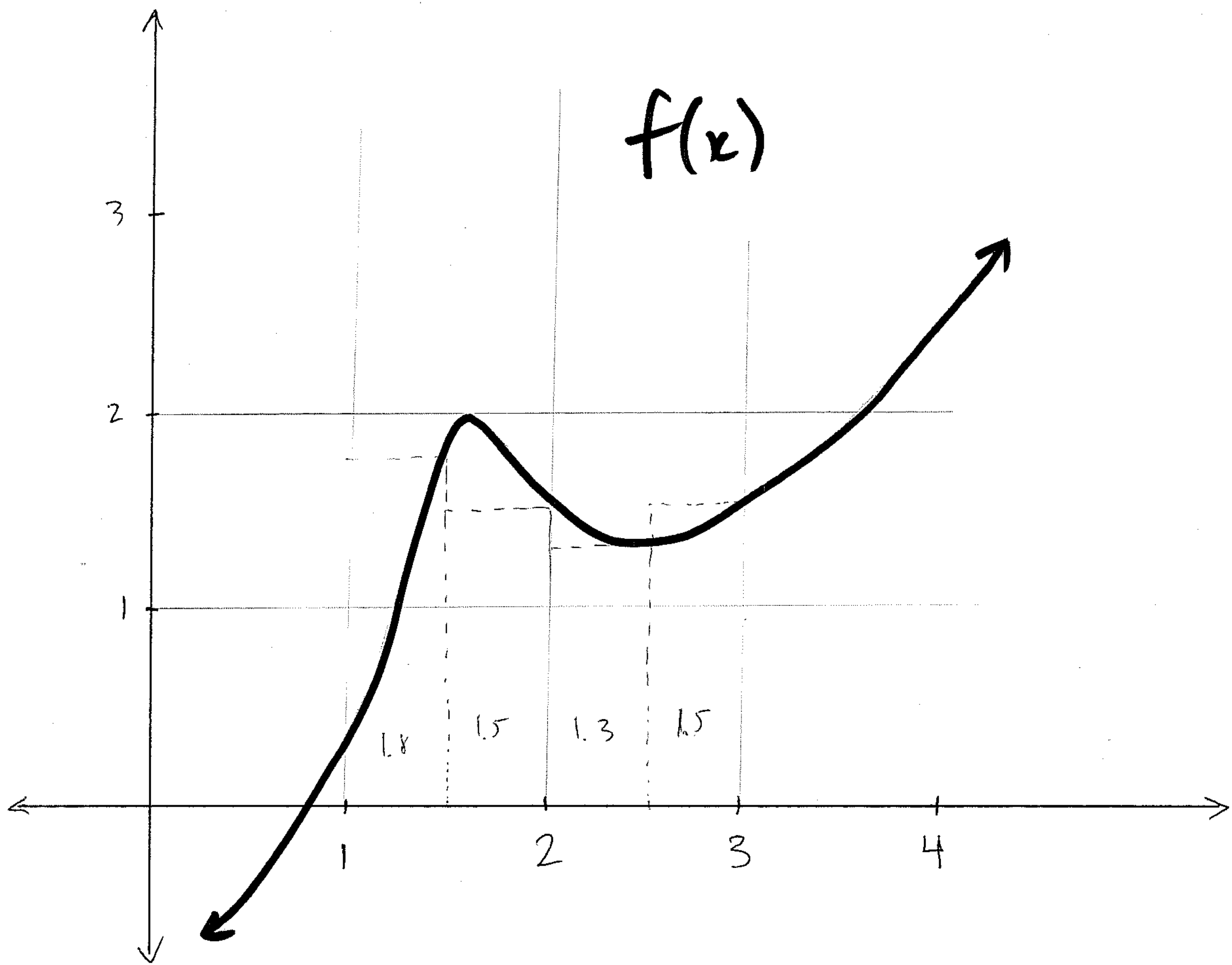
$$= \int \left(\frac{1}{x} + x^4 + 3x^{-3} \right) dx$$

$$= \boxed{\ln|x| + \frac{x^5}{5} + \frac{3}{-2}x^{-2} + C}$$

Extra Credit(1 point) Calculate the definite integral: $\int_e^\pi e^\pi \, dx$

$$e^\pi \cdot x \Big|_e^\pi = \pi e^\pi - e \cdot e^\pi \quad \text{or} \quad \pi e^\pi - e^{\pi+1}$$

$$= \boxed{(\pi - e) e^\pi}$$


f(x)
3
2
1
1
2
3
4
1.8
1.5
1.3
1.5

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